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ODD–EVEN STAGGERING IN THE YIELDS OF INTERMEDIATE MASS FRAGMENTS FROM p+Ag COLLISIONS AT $Ep = 480$ MeV

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The experimental total production cross sections of intermediate mass fragments (isotopes of Li, Be, B, C, N, O, F, Ne, Na, and Mg) were extracted by integration of $\mathrm{d}^2\sigma/\mathrm{d}\Omega\ \mathrm{d}E$ data measured at several angles for p+Ag collisions at proton beam energy of 480 MeV. The total cross sections show typical odd–even staggering (OES) when presented as a function of the atomic number Z of ejectiles. The effect is the strongest for products with $N = Z$ and $N = Z + 2$. Similar behaviour is observed for theoretical cross sections evaluated in the two-step model in which the first stage of the reaction is described by intranuclear cascade INCL++ and the second stage by GEMINI++ model or by two other models, namely ABLA07 and SMM. The OES seems to be even more pronounced for theoretical than for the experimental cross sections.

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1. Introduction

The odd–even staggering (OES), *i.e.* enhanced production yield of even-Z products in respect to neighbouring odd-Z ones (or *vice versa*) has been observed in many spallation and fragmentation reactions, *e.g.*, [1–6]. The

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origin of this effect is not completely understood but it is usually attributed to the structural effects (pairing and shell closure) influencing the density of states of the nuclei available in the final step of the reaction processes.

Spallation reactions induced by high-energy protons in collisions with atomic nuclei have attracted attention of physicists since over a half of the century. Such an interest is caused by a very broad range of subjects in which these reactions play a significant role. This concerns both, *(i)* physical phenomena as, *e.g.* interaction of cosmic rays with the interstellar matter composed mainly of protons which leads to the modification of nuclear content of observed cosmic rays in respect to the content at the source of these rays [7] and *(ii)* multiple applications of the spallation reactions in the science and technology. They cover such problems as projecting and building of efficient neutron sources (so-called spallation sources) [8, 9] which are being used in numerous solid state physics investigations [10] as well as may be applied in accelerator-driven fission reactors, to change long-living radioactive isotopes of the nuclear waste into isotopes of much shorter lifetime [11], for production of exotic nuclei [12], *etc.* Such a broad range of applications of the spallation reactions demands the knowledge of the cross sections for production of various nuclides, frequently in interaction of protons with unstable and short-living atomic nuclei. In this case, it is difficult, time consuming, and/or very expensive to obtain experimentally a desirable information. Therefore, an existence of a reliable theory of the mechanisms contributing to the spallation reactions is necessary. Unfortunately, the present day status of the nuclear reaction theory does not allow to solve exactly such a multibody nuclear problem. Thus, in practice, various simplified models of the spallation reactions are proposed.

It is, therefore, obvious that a validation of the models is necessary to check which of them is appropriate for realistic and efficient description of the total and differential cross sections of the spallation reactions. This is usually performed by comparing values of the isotopically identified experimental cross sections with predictions of various theoretical models. For example, such a validation was done for total cross sections of $p+^{136}\mathrm{Xe}$ collisions in Refs. [13] and [14] for proton beam energy of 0.5 GeV and 1.0 GeV, respectively.

The ability of the models to reproduce the staggering effect in the yields of the spallation reactions seems to be very demanding condition which requires not only the reproduction of the smooth variation of the total cross sections *versus* atomic number Z and mass number A but also reproduction of details of its change from even to odd values of these numbers. Thus, the study of the staggering effect seems to be interesting and important subject for the investigation of the reaction mechanism.

It is known that the OES is most pronounced for light products with $Z = N$, decreases with increase of their $N - Z$ difference as well as with increase of the atomic number Z of the ejectiles [6]. Therefore, this effect should be very well visible in the emission of intermediate mass fragments (IMF), *i.e.* the particles heavier than ^{4}He and lighter than the fission fragments. In the present work, the analysis of data measured by Green *et al.* [15] for p+Ag reaction at proton beam energy 480 MeV has been performed with the aim to study experimentally the OES as a function of the atomic number Z of emitted intermediate mass fragments and to compare the experimental results with predictions of widely used theoretical models. Since the data do not contain the total production cross sections but double differential cross sections $\mathrm{d}^2\sigma/\mathrm{d}\Omega\ \mathrm{d}E$, some procedure has to be applied for determination of the total production cross sections. This subject is presented in the second section of the paper and the staggering of the cross sections is then qualitatively discussed.

The third section deals with the quantitative estimation of the staggering of experimental cross sections as well as of the theoretical cross sections calculated according to the two-stage model of the reaction. The first stage was reproduced by the INCL++ model [16], whereas the second stage by three different models: ABLA07 [17], GEMINI++ [18, 19], and SMM [20]. These models describe emission of the intermediate mass fragments from equilibrated remnant nuclei of the first stage of the reaction which is treated as intranuclear cascade of nucleon–nucleon and pion–nucleon collisions.

In the following section, the sensitivity of the staggering effect to the non-equilibrium emission of fragments from the first stage of the reaction is discussed. Here, influence of the coalescence of nucleons into light charged particles (LCP), *i.e.* the isotopes of H and He with mass number not larger than $A = 4$ and into intermediate mass fragments is considered.

The results are summarized and discussed in the last section of the paper.

2. Experimental total production cross sections

The differential cross sections $\mathrm{d}^2\sigma/\mathrm{d}\Omega\ \mathrm{d}E$ of Green *et al.* [15] were measured for p+Ag collisions at proton beam energy of 480 MeV for several scattering angles between 10° and 160°. The intermediate mass fragments, *i.e.* the isotopes of Li, Be, B, C, N, O, F, Ne, Na and Mg were detected. The spectra for lightest products (Li and Be isotopes) were measured at 9 different angles, those for heavier products at 6 different angles and several heaviest isotopes only at 5 or even 4 angles. All the spectra have smooth energy and angular dependence. This allowed to extract total cross sections by integration over the angle and the energy. For this purpose, procedures for interpolation and extrapolation of the cross sections for different emission angles have to be applied.

All the spectra are isotropic for small energies of the observed particles (smaller than approx. 30 MeV), however, at higher energies the spectra are forward peaked. Such characteristic behaviour of the data may be interpreted as the indication of two different mechanisms of the reaction. First mechanism corresponds to the two-step model which assumes that the first, fast stage of the reaction consists in intranuclear cascade of the nucleon–nucleon and nucleon–pion collisions. During this fast stage of the process, mainly nucleons and pions are emitted leading to fast equilibration of the excited remnant nucleus. The second stage of the reaction consists in the de-excitation of the equilibrated residual nucleus through almost isotropic emission of various particles — neutrons, light charged particles as well as intermediate mass fragments. Observation of anisotropic emission of complex, energetic particles seems to indicate that they originate from the first stage of the reaction.

The INCL++ [16] — the most popular model of the first step of the reaction — treats the process as the intranuclear cascade of the nucleon–nucleon and pion–nucleon collisions. However, it allows besides the emission of pions and nucleons also for emission of light charged particles and intermediate mass fragments. This process is described as the coalescence into the composite particles of the nucleons which are close enough in the coordinate and momentum space in respect to the escaping nucleon. It was observed that this surface coalescence model works well for light charged particles [21] and to some extend also for intermediate mass fragments [22] but the maximal mass number A of considered IMF has to be, in practice, limited to values not larger than 8 due to a very long computing time.

In the present study, the INCL++ model (version 5.3) of the intranuclear cascade has been used. A possibility of the emission of complex light charged particles in this stage of the process has been taken into account to assure achieving a realistic mass, charge and excitation energy distribution of the residual compound nuclei. However, the coalescence of the nucleons escaping from the intranuclear cascade with creation of intermediate mass fragments was not allowed because of two reasons: *(i)* the INCL++ enables one to perform efficiently such calculations only for the lightest IMF and *(ii)* it was found in the earlier study of these reactions [23] that high-energy spectra of IMF are significantly overestimated by this model. It is important to emphasize that the above decision does not modify significantly the mass, charge and energy distribution of the excited nuclei — the remnants of the cascade since the cross sections for production of IMF are orders of magnitude smaller than those for nucleons and complex LCP. The second stage of the process, *i.e.* emission of particles from the excited compound nuclei — residuals of the fast stage of the reactions was described by GEMINI++ model [18]. It was shown in Ref. [23] that the use of INCL plus GEMINI models enables one to reproduce to large extend the experimental spectra and their energy dependence for heaviest products of the p+Ag reaction analysed in the present study. Furthermore, it was found that these two models always underestimate the differential cross sections for light intermediate mass fragments, *i.e.* they leave the room for contribution of an additional mechanism. This contribution must be determined phenomenologically by adjusting some free parameters.

To obtain the total production cross section, the GEMINI++ double differential cross sections $\mathrm{d}^2\sigma/\mathrm{d}E\,\mathrm{d}\Omega$ were supplemented by incoherently added isotropic emission from highly excited Maxwellian source (or two sources) moving along the beam direction. The parameters of the source, *i.e.* its velocity β, apparent temperature T, the contribution to the total cross section σ and the parameters responsible for the Coulomb barrier hindering the emission of ejectiles from the source were fitted to reproduce simultaneously the spectra of given ejectile at all scattering angles. Details of the moving source model as well as the interpretation of its parameters can be found in Appendix of Ref. [24].

Very good reproduction of most of the data was achieved using one moving source contribution. Only 10 lightest IMF among all 39 studied particles, *i.e.* ^{6,7}Li, ^{7,9,10}Be, ^{10,11,12}B and ^{11,12}C needed application of two moving sources for the good reproduction of energy spectra at all investigated scattering angles from 20° to 160°.

The procedure described above enabled us to obtain non-equilibrium production cross section of IMF equal to the parameter σ_1 of the moving source (or to the sum of σ_1 and σ_2 — the appropriate parameters of both moving sources). Furthermore, sum of the equilibrium production cross section evaluated by means of GEMINI++ and the above non-equilibrium cross section provided value of the total production cross section.

Total production cross sections are presented in Fig. 1 as a function of the atomic number of appropriate IMF. A very pronounced staggering of the cross sections is visible for products with even mass number A (upper part of the figure). Weaker effect appears for odd A products with $N = Z + 1$ and almost no effect is present for those with $N = Z + 3$ (lower part of the figure).

To discuss quantitatively the staggering effect of data as well as to compare it with predictions of the models, one needs to introduce some variable whose value would give the needed information. This is done in the next section.

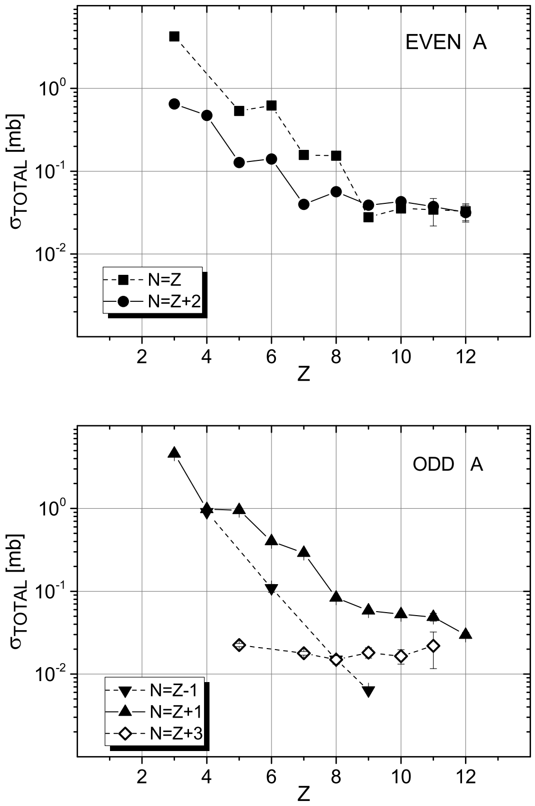

Fig. 1. Plot of the total cross sections for emission of intermediate mass fragments from p+Ag collisions at 480 MeV proton beam energy. The mass number A, neutron number N and atomic number Z correspond to appropriate IMF.

3. Quantitative analysis of the staggering

To determine quantitatively the OES effect, we applied the procedure proposed by Tracy *et al.* [25] in the form given by Ricciardi *et al.* [3] for Z dependence of the cross sections at fixed $N-Z$. The relative enhancement of the cross sections for even-Z (odd-Z) products in respect to smooth Z dependence is given by larger than zero (smaller than zero) value of the δ function which is calculated according to the formula

$$\delta\left(Z+3/2\right) \equiv \tfrac{1}{8}\left(-1\right)^{Z+1}\left[\left(L_3-L_0\right)-3\left(L_2-L_1\right)\right] , \qquad (1)$$

where

$$L_i \equiv \ln\left(\sigma(Z+i)\right) .$$

The δ value is attributed to the centre of the Z interval from Z to $Z+3$ in which the smooth Z dependence was postulated.

In Fig. 2, the δ function is presented for experimental total cross sections (open black squares) and for theoretical cross sections (full coloured symbols) evaluated in the two-step model in which the INCL++ is coupled to three different models (ABLA07, GEMINI++ and SMM) describing the de-excitation of the equilibrated, excited remnant nucleus from the intranuclear cascade. The surface coalescence of the nucleons escaping from the intranuclear cascade was included for LCP as it was done during procedure of estimation of the total experimental cross sections.

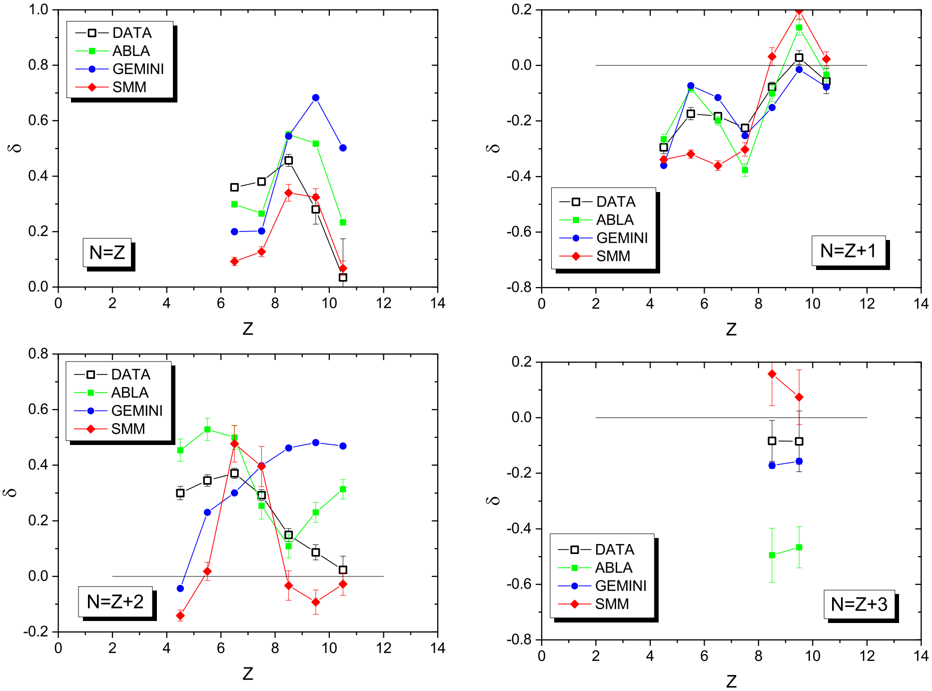

Fig. 2. (Color online) Plot of the δ function *versus* atomic number Z of the reaction products. The open, black squares depict values of the δ function evaluated for experimental cross sections, whereas full green squares, blue dots, and red diamond symbols correspond to the δ-function values obtained with the cross sections of ABLA07, GEMINI++ and SMM models, respectively. The horizontal lines in the right panels and in the left, lower panel of the figure represent the $\delta = 0$ axis.

As can be seen in Fig. 2, the delta-function values are positive for experimental cross sections of IMF with $N = Z$ (left, upper panel of the figure) and with $N = Z + 2$ (left, lower panel), whereas they are negative for IMF with $N = Z{+}1$ (right, upper panel) and $N = Z{+}3$ (right, lower panel). This means that the even-Z cross sections are larger than the smooth trend for $N = Z$ and $N = Z{+}2$, but odd-Z cross sections are larger for $N = Z{+}1$ and $Z + 3$. This information agrees perfectly with qualitative conclusions from inspection of Fig. 1, where the experimental cross sections are collected.

Furthermore, absolute values of the δ function are close to zero for products with $N = Z + 3$ but are significantly different from zero for other products. This again agrees with information derived from Fig. 1, where Z dependence of the cross sections is quite smooth for particles with $N = Z + 3$ and is sawtooth-like for other particles, especially those with $N = Z$ and $N = Z + 2$. Moreover, the δ-function values for experimental cross sections become closer to zero with increasing atomic number Z of the products what agrees with the general tendency of decreasing of the OES effect for larger Z (*cf.* Ref. [6]).

In spite of the general agreement between the sign of the δ function of experimental cross sections with those of all three models, there are also visible deviations between experimental and theoretical δ functions. They are especially distinct for products with $N = Z + 2$. In this case, the δ function evaluated for experimental cross sections *decreases* from values around 0.3 at $Z = 4.5$ to zero at $Z = 10.5$, whereas the δ function determined for GEMINI++ cross sections monotonically *increases* from -0.05 at $Z = 4.5$ to 0.47 at $Z = 10.5$. Thus, even the tendency of variation of the experimental δ function and that of the GEMINI++ is not reproduced. Distinct differences between experimental and model δ functions exist also for ABLA07 (the δ-function values do not change monotonically) and for SMM (δ-function values are large and positive only for two points in the 4.5–10.5 interval, whereas they are small and negative at both ends of this interval).

Smaller but also distinct differences between experimental and model δ functions are present for reaction products with $N = Z$. The shape of all model δ functions is almost the same — similar to that of the experimental δ function but there are two differences. The magnitude of experimental and model values of the δ function is different and position of the maximum of model δ function is shifted in respect to that of the experimental one.

The question arises what is the origin of the differences between the δ function determined from the experimental cross sections and that which was evaluated from model cross sections calculated in the frame of the two-step model with different models applied for description of the second stage of the reaction.

The natural candidate for explanation of this differences seems to be neglecting the non-equilibrium processes in the model calculations. They were introduced in a phenomenological way in the total experimental cross sections but they are not explicitly present in the model cross sections. They are only included into evaluation of the cross sections of light charged particles, which are not analyzed in the present study. Of course, they influence the population of residual nuclei after the intranuclear cascade because the coalescence of nucleons into ejectiles composed of less than 5 nucleons. Furthermore, such coalescence process modifies the population of the excited states of residuals of the cascade.

To check whether an increasing of the coalescence of nucleons during the stage of the intranuclear cascade may significantly modify the δ function evaluated from model cross sections, we repeated the model calculations extending the coalescence effect up to intermediate mass fragments with mass number A equal to 8. It turned out that such a modification had no significant influence.

4. Summary

In the present work, the OES, *i.e.* the effect of the staggering in the yields of intermediate mass fragments produced in p+Ag collisions at proton beam energy of 480 MeV has been studied. Since the total production cross sections were not measured for this reaction, they were determined by integration of the experimental double differential cross sections $\mathrm{d}^2\sigma/\mathrm{d}\Omega\ \mathrm{d}E$ of Ref. [15]. These double differential cross sections were analyzed by combination of the two-step model of the intranuclear cascade INCL++ followed by the de-excitation of the equilibrated remnant nucleus of the cascade in the frame of the GEMINI++ model. The difference between predictions of this model and the experimental data were fitted by means of the phenomenological model of one or two isotropically emitting sources moving forward (along the beam direction). Such a method provided model-independent values of the experimental cross sections because combination of the above models can be treated as a mean for appropriate interpolation and extrapolation of the data for the full angular and energy range necessary for trustworthy result of the angle and energy integration.

The obtained total cross sections were analyzed qualitatively as a function of atomic number Z of the products as well as quantitatively by application of the δ function proposed by Tracy *et al.* [25] for determination of the relative enhancement of the cross sections for even-Z (odd-Z) products in respect to their smooth Z dependence. It was found that a significant OES effect is present, most pronounced for products with $N = Z$, $N = Z + 1$ and $N = Z+2$. In the first and the third case, the even-Z products are produced with relatively larger cross sections than the smooth Z dependence, whereas in the second case, this was true for odd-Z products.

The staggering of the experimental total cross sections was compared with that observed for the theoretical cross sections of the two-step model. The first stage of the reaction was described by the INCL++ model with inclusion of coalescence of escaping nucleons from the intranuclear cascade into complex nuclei with mass number A smaller than 5 and, separately, with mass number A smaller than 9. In both cases, the second stage of the process, *i.e.* the de-excitation of the equilibrated nucleus remnant of the intranuclear cascade was modelled by three different programs: ABLA07, GEMINI++ and SMM. It was found that the main properties of the experimental cross section staggering, *i.e.* the sign of the δ function as well as its average value was reproduced by all three models. However, quite significant differences appeared as concerns the details of the staggering, *i.e.* the shape of the Z dependence of the δ function and to smaller extent its values. The strongest differences were observed for $N = Z + 2$ nuclei.

The origin of the difference between the data and model predictions might be attributed to the lack of knowledge of non-equilibrium processes which seem to be quite important for the studied reactions [23]. However, it was checked that the δ function does not practically change when the coalescence of nucleons from the intranuclear cascade into complex ejectiles was extended from particles with $A < 5$ to those with $A < 9$ for all models of the second step of the reaction. This seems to indicate that the possible non-equilibrium processes appearing in the first stage of the collision process do not influence strongly the staggering.

It should be pointed out that the modification of the δ function by using different models of the second stage of the reaction is at least as large as the difference between the experimental and model δ functions. This might be interpreted as strong dependence of the predicted OES on the physical assumptions underlying the theoretical models.

In summary, the present results indicate that the OES seems to be a demanding effect for testing the assumptions underlying the theoretical models of the spallation reactions and, therefore, its investigation may be helpful in their development.

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