STUDIES OF MESIC NUCLEI VIA DECAY REACTIONS*

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Collisions in a system of two particles at energies close to a bound state in different channels are discussed. Next, the bound state decays into a third coupled channel. A phenomenological approach to $dd \to \pi^- p\ {}^3\mathrm{He}$ reaction is presented.

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1. Introduction

The possibility of η-nuclear quasi-bound states was first discussed by Haider and Liu [1, 2] a long time ago. The existence of such states has been elusive, however. At this moment, the experimental evidence is rather indirect, getting the most clear indication from the measurements of $pd \to \eta^3\mathrm{He}$ [3, 4] and of $dp \to \eta^3\mathrm{He}$ [5, 6], and from the realization [7] that the rapid slope of the cross section close to threshold may be a signal of a quasi-bound state. The same behavior of the total cross section was also confirmed in the photon induced reaction $\gamma^3\mathrm{He} \to \eta^3\mathrm{He}$ [8]. The slope indicates large scattering length, but the final state η^3He interaction does not allow to determine the sign of this length which would demonstrate that either a bound state or a virtual state is observed. Additional information is necessary. One possibility is to use the (π, η) reaction on a three-nucleon target. Such an analysis indicates that the η^3He system is not a bound but a virtual state [9].

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Analogical enhancement close to the kinematic threshold was observed in the total cross section of the $dd \to {}^4\mathrm{He}\eta$ reaction [10–12]. Again, these results suggest a large scattering length, however, do not give a conclusive answer whether the bound state exists.

Having the scattering length A, one may extrapolate the scattering matrix

$$T = \frac{A}{1 - ipA}\,, \tag{1}$$

where p is the η–He relative momentum at some distance below the threshold. In this region, $p = \sqrt{2\mu E}$ becomes complex, $p = i|p|$. For Real $A > 0$, one obtains the zero of the denominator and the singularity of the T matrix on the physical sheet. That means a bound state for which Im $A \neq 0$ becomes a quasi-bound state. In the case of Real $A < 0$, the zero of denominator may also happen but for $p = -i|p|$, it lays on the second Riemann sheet of the complex energy plane. Such a state is called virtual and makes an analogy of nucleon–nucleon, spin-0, isospin-1 state known as anti-deuteron (named so because of the opposite sign of the pole position in the Im p axis). Going some distance below the threshold (usually a short distance, as A depends on energy and equation (1) with constant A looses its applicability), one may notice different behavior of $|T|$ in both cases. For a bound or quasi-bound state, $|T|$ grows up until the energy of the bound state is reached. On the other hand, in the case of a virtual state $|T|$ drops down immediately below the threshold.

Direct observation of the elastic scattering amplitude below the threshold is not feasible. However, one can observe a similar behavior in channels coupled to the channel where the bound state is suspected to exist. Thus, in the case of reaction

$$dd \to \pi^- \, p \, {}^3\mathrm{He} \tag{2}$$

the channel of interest is $\eta\ {}^4\mathrm{He}$ and the decay channel consists of three particles π, p and ${}^3\mathrm{He}$.

As the $\eta\ {}^3\mathrm{He}$ system seems almost bound, the $\eta\ {}^4\mathrm{He}$ system is likely to be bound. The $\pi^-\ p\ {}^3\mathrm{He}$ might be expected to be the dominant decay channel. In such circumstances, one could expect a subthreshold enhancement in the cross section for reaction (2). Surprisingly, there is no experimental confirmation of such an effect. Measurements [13–16] offer a cross section of about 200 nb which is apparently due to a quasi-free reaction. An upper limit of the fraction that proceeds through a quasi-bound state is obtained at a level of 25 nb [14].

The aim of this work is to calculate/estimate the magnitude of

$$dd \to \left(\eta\ {}^4\mathrm{He}\right)_{\mathrm{bound}} \to \pi^- \, p \, {}^3\mathrm{He} \tag{3}$$

reactions and to offer some speculations on the existence of the $(\eta\ {}^4\mathrm{He})_{\mathrm{bound}}$ state.

2. Cross sections for the bound state formation and decays

2.1. Approximate amplitude for a two body process

Consider transition of two initial particles denoted by D, D' into two particles B, B' of a higher mass threshold. Particles BB' are assumed to form an unstable, S-wave, bound state $|B\rangle$ of energy E_B and width Γ. There may be several modes of decay of this bound state and corresponding partial widths are denoted by Γ_i.

The reaction of interest consists of three steps:

- colliding D, D' particles generate unstable state $|B\rangle$,

• • unstable state $|B\rangle$ lives for some time and

• • • the unstable state $|B\rangle$ decays into state $|F\rangle$.

In this section, all these stages of reaction are described in a phenomenological way. The formulation used below is general but some approximations are made for a specific case: $D = D' =$ deuteron, $B =$ ^{4}He and $B' = \eta$.

• It is assumed that the basic initial reaction

$$D + D' \to B + B' \tag{4}$$

has been studied experimentally in some region above the BB' threshold. The relevant cross section σ_{DB} in the threshold region may be presented in the form

$$\sigma_{DB} = S(p_B)\, p_B \,, \tag{5}$$

where the threshold behavior is described in part by p_B — the relative momentum in the BB' channel. The function $S(p_B)$ is to be extracted from experiment. With deeply bound or broad states $S(p_B)$ is a weakly energy dependent function, for weak binding, it may indicate a sharp threshold peak.

This cross section is generated by an operator V_{DB} which in a standard way allows to calculate the related scattering amplitude f_{DB}

$$f_{DB} = \frac{2\mu_{BB'}}{4\pi} \langle DD|V_{DB}|B, p_B\rangle \,. \tag{6}$$

$\mu_{BB'}$ is the reduced mass and p_B, p_D are the relative momenta in the corresponding channels. The cross section becomes

$$\frac{d\sigma_{DB}}{d\Omega} = |f_{DB}|^2 \, \frac{p_B}{\mu_{BB'}} \, \frac{\mu_{DD}}{p_D}. \tag{7}$$

A difficulty arises at this stage: from the scattering experiments, one can extract $|\langle DD|V_{DB}|B, p_B\rangle|$ which is the modulus of the on-shell transition amplitude for a given momentum p_B, while one needs the transition to the bound state $\langle DD|V_{DB}|B, E_B\rangle$. Formally,

$$\langle DD|V_{DB}|B, E_B\rangle = \int d\boldsymbol{p}_B \langle DD|V_{DB}|B, p_B\rangle \langle p_B, B|B, E_B\rangle \,, \tag{8}$$

where $\langle p_B, B|B, E_B\rangle = \Psi_{BB'}(p_B)$ is the wave function of the bound state in the momentum space. Equation (8) involves integration over all momenta p_B and not only over the momenta allowed by the energy conservation. To proceed without a specific model of V_{DB}, we assume that the spacial range of this operator is characterized by the size of the final particle B (that is ^{4}He in the case of interest). This is the basic approximation of this calculation,

$$\langle DD|V_{DB}|B, p_B\rangle = |C_{DB}|\Psi_B(p_B)\,, \tag{9}$$

where $\Psi_B(p_B)$ is the profile of single nucleon wave function in nucleus B folded over η-nucleon interaction range and $|C_{DB}|$ is a constant determined from the slope of the cross section. For an estimate, we use a Gaussian

$$\Psi_B(p) = \exp\left(-\frac{R_B^2 p^2}{2}\right) \tag{10}$$

and to simplify the estimate, we assume the bound state wave function in the same form

$$\Psi_{BB'}(p) = \exp\left(-\frac{R_{BB'}^2 p^2}{2}\right)\left[\frac{R_{BB'}^2}{\pi}\right]^{3/4}. \tag{11}$$

• • The propagation of the bound state is described by

$$G_{BB'} = \frac{\Psi_{BB'}^*(p)\Psi_{BB'}(p)}{E - E_B + i\Gamma/2}\,, \tag{12}$$

where the complex part of the energy corresponds to the total decay rate.

• • • Decay of the $|B\rangle$ state into final $|F_{\mathrm{i}}\rangle$ state is given by an operator V_{BF}. The matrix element of this operator between the bound and the final state $\langle B|V_{BF}|F_{\mathrm{i}}\rangle$ determines the partial width of the state. The Fermi formula gives

$$\frac{\Gamma_i}{2} = 2\pi\int d\boldsymbol{p}|\langle B|V_{BF}|F_{\mathrm{i}}\rangle|^2\delta(E - E_F(p)) = (4\pi)^2 p_F\ \mu_{FF'}|\langle B|V_{BF}|F_{\mathrm{i}}, p_F\rangle|^2 \tag{13}$$

from which we obtain

$$|\langle B|V_{BF}|F_{\mathrm{i}}, p_F\rangle|^2 = \frac{\Gamma_i}{2p_F\mu_{FF'}(4\pi)^2}\,. \tag{14}$$

In this calculation, it is assumed that ^{3}He is a spectator in the decay process and the final decay energy is carried by the meson and the proton. For simplicity, the non-relativistic phase space is used. This may look suspicious in the π-meson case but the relevant reduced mass drops out from the final expression of the cross section. This calculation may be easily improved anyway. Here, it serves also as a check of the normalization used.

2.2. Estimates of the cross section

The transition matrix element for the process in question is given by

$$\langle D|V_{D\to B\to F}|F_{\rm i}\rangle = \int d\boldsymbol{q}\langle D|V_{DB}|Bq\rangle \frac{\langle q|B,E_B\rangle\langle B,E_B|V_{BF}|F_{\rm i}\rangle}{E-E_B+i\Gamma/2}\,. \tag{15}$$

The related scattering amplitude is as in equation (6)

$$f_{D\to B\to F} = \frac{2\mu_{FF'}}{4\pi}\langle D|V_{D\to B\to F}|F_{\rm i}\rangle \tag{16}$$

and the cross section

$$\frac{d\sigma_{DB}}{d\Omega} = |f_{DB}|^2\,\frac{p_B}{\mu_{BB'}}\,\frac{\mu_{FF}}{p_F}\,. \tag{17}$$

Collecting all factors and approximations, the formula

$$\sigma_{DF} = \frac{\sigma_{DB}}{p_B}\,\frac{\sqrt{\pi}}{16}\,\frac{\Gamma^\pi}{(E-E_B)^2+(\Gamma/2)^2}\frac{1}{\mu_{BB'}R^3} \tag{18}$$

is obtained where the two radii were set equal $R_B = R_{BB'} \equiv R$.

The factor $\frac{\sigma_{DB}}{p_B} \mapsto C_{BD} \simeq 0.3$ nb/[MeV/c] may be obtained from experimental cross sections measured and collected in Ref. [17]. With the expected values $\Gamma^\pi \simeq 10$ MeV, $\Gamma \simeq 20$ MeV and $R \simeq 2.5$ fm, one obtains $\sigma_{DF} \simeq 4.5$ nb at the peak. The result is most sensitive to the radius R but the numbers obtained are below the experimental limit. However, the relation to the actual experimental limit is not that straightforward. It is the interference of reaction (3) with the quasi-free reaction that generates the experimental limit of 25 nb. This requires specific models and phase relations. At this moment the question:

3. Are there η ^{4}He bound states?

cannot be fully answered neither by experiment nor by theory. Simple, old calculations of the threshold behavior in η ^{3}He and η ^{4}He systems [18] indicated a η ^{4}He bound state. Since then two basic ingredients have changed:

- better understanding of two nucleon $\eta\, NN \to NN$ decay mode,
- a better knowledge of the subthreshold ηN scattering amplitude.

The latter is represented by a best fit to multi-channel scattering data obtained in Ref. [19] and plotted in Fig. 1. An average energy involved in the ηN center-of-mass amounts to -36 MeV (21 MeV binding and about 14 MeV of the residual nucleus recoil). So far, below the threshold the absorptive part of the amplitude is fairly small and the rate of $\pi^- p$ decay

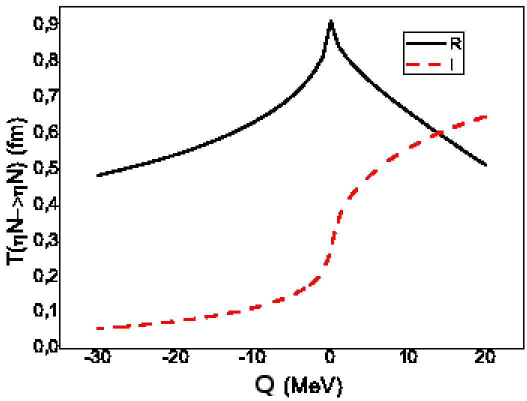

Fig. 1. The elastic η–N scattering amplitude plotted against the c.m. kinetic energy Q. Real part — continuous line, absorptive part — dashed line.

might be strongly reduced. That reduces the chance of observation via the reaction (3). This plot shows also that the attractive nuclear potential related to Re $T_{\eta N}$ may be weaker than in the $\eta\ ^3$He case which involves about -12 MeV subthreshold extrapolation.

On the other hand, the decay of the $\eta\ ^4$He bound state into two nucleon mode may be strongly enhanced. A phenomenological evaluation of the rates is possible as the cross sections for

$$pp \to pp\eta\,, \tag{19}$$

$$pn \to d\eta\,, \tag{20}$$

$$pn \to pn\eta \tag{21}$$

have been measured in the close to threshold region [21, 22]. The analysis based on the detailed balance corrected for the final state interaction has been performed in Ref. [20]. At central nuclear densities, the related absorptive potential of the $\rho(r)^2$ profile with a strength Im $W_{NN}(r=0) = 3.2$ MeV was obtained. However, helium nucleus is twice as dense and the corresponding absorptive potential rises to Im $W_{NN}(r=0) \simeq 13$ MeV. Such strong absorption may prevent binding or lead to much larger level width. To resolve some of the problems, it would be useful to have also measurements of another

$$DD \to \left(\eta\ ^4\mathrm{He}\right)_{\mathrm{bound}} \to pn\, D \tag{22}$$

decay process.

From the experimental field, the ongoing analysis of the reactions $dd \to\ ^3\mathrm{He}\, p\pi^-$ and $dd \to\ ^3\mathrm{He}\, n\pi^0 \to\ ^3\mathrm{He}\, n\gamma\gamma$ from WASA-at-COSY, which will reach the sensitivity of several nb [16], should help to answer the question of the existence of the bound state.

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