Combined analysis of the K^+K^- interaction using near-threshold $pp \to ppK^+K^-$ data

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The K^+K^- final state interaction was investigated based on both the K^+K^- invariant mass distributions measured at excess energies of $Q = 10$ and 28 MeV and the near threshold excitation function for the $pp \to ppK^+K^-$ reaction. The K^+K^- final state enhancement factor was parametrized using the effective range expansion. The effective range of the K^+K^- interaction was estimated to be $\mathrm{Re}(b_{K^+K^-}) = -0.1 \pm 0.4_{\mathrm{stat}} \pm 0.3_{\mathrm{sys}}$ fm and $\mathrm{Im}(b_{K^+K^-}) = 1.2^{+0.1_{\mathrm{stat}}+0.2_{\mathrm{sys}}}_{-0.2_{\mathrm{stat}}-0.0_{\mathrm{sys}}}$ fm, and the determined real and imaginary parts of the K^+K^- scattering length amount to $|\mathrm{Re}(a_{K^+K^-})| = 8.0^{+6.0_{\mathrm{stat}}}_{-4.0_{\mathrm{stat}}}$ fm and $\mathrm{Im}(a_{K^+K^-}) = 0.0^{+20.0_{\mathrm{stat}}}_{-5.0_{\mathrm{stat}}}$ fm.

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I. INTRODUCTION

The strength of the K^+K^- interaction is a crucial quantity regarding the formation of a hypothetical kaon–antikaon bound state. Existence of such a state could explain the nature of the $a_0(980)$ and $f_0(980)$ scalar mesons [1,2], whose masses are very close to the sum of the K^+ and K^- masses.[1] Among many theoretical investigations [7–11] the K^+K^- interaction was studied also experimentally in the $pp \to ppK^+K^-$ reaction with COSY-11 and ANKE detectors operating at the COSY synchrotron in Jülich, Germany [12–18]. The experimental data collected systematically below [12–16] and above [18] the ϕ meson threshold reveal a significant enhancement in the shape of the excitation function near the kinematical threshold, which may be due to the final state interaction (FSI) in the ppK^+K^- system. The indication of the influence of the pK^- final state interaction was found in both COSY-11 and ANKE data in the ratios of the differential cross sections as a function of the pK and the ppK invariant masses,

$$R_{pK} = \frac{d\sigma/dM_{pK^-}}{d\sigma/dM_{pK^+}}, \quad R_{ppK} = \frac{d\sigma/dM_{ppK^-}}{d\sigma/dM_{ppK^+}},$$

where a significant enhancement in the region of both the low pK^- invariant mass M_{pK^-} and the low ppK^- invariant mass M_{ppK^-} is observed [15,19]. The phenomenological model based on the factorization of the final state interaction into interactions in the pp and pK^- subsystems, neglecting the K^+K^- potential, does not describe the whole experimental excitation function for the $pp \to ppK^+K^-$ reaction, underestimating the data very close to the kinematical threshold [15,20]. This indicates that in the low-energy region the influence of the K^+K^- final state interaction may be significant [15,19,20]. Motivated by this observation the COSY-11 Collaboration has recently estimated the scattering length of the K^+K^- interaction based on the $pp \to ppK^+K^-$ reaction measured at excess energies of $Q = 10$ and 28 MeV [19]. As a result of the analysis the K^+K^- scattering length was determined based on the low-energy proton-proton (M_{pp}) and K^+K^- (M_{KK}) invariant mass distributions (so-called Goldhaber plot) shown in Fig. 1 [19].

In this article we combine the Goldhaber plot distribution established by the COSY-11 group with the experimental excitation function [12–15,18] near threshold and determine the K^+K^- scattering length with better precision compared to the previous results. We have also extracted the effective range of the K^+K^- interaction.

II. DESCRIPTION OF THE FINAL STATE INTERACTION IN THE ppK^+K^- SYSTEM

As in the previous analysis [19] we use the factorization ansatz proposed by the ANKE group with an additional term describing the interaction in the K^+K^- system. We assume that the overall enhancement factor originating from final state interaction can be factorized into enhancements in the proton-proton, the two pK^-, and the K^+K^- subsystems:[2]

$$F_{FSI} = F_{pp}(k_1) \times F_{p_1K^-}(k_2) \times F_{p_2K^-}(k_3) \times F_{K^+K^-}(k_4), \quad (1)$$

where k_j stands for the relative momentum of particles in the corresponding subsystem [19]. The proton-proton scattering amplitude was taken into account using the following parametrization:

$$F_{pp} = \frac{e^{i\delta_{pp}(^1\mathrm{S}_0)} \sin\delta_{pp}(^1\mathrm{S}_0)}{Ck_1},$$

where C stands for the square root of the Coulomb penetration factor [21]. The parameter $\delta_{pp}(^1\mathrm{S}_0)$ denotes the phase shift calculated according to the modified Cini-Fubini-Stanghellini formula with the Wong-Noyes Coulomb correction [22–24]. Factors describing the enhancement originating from the interaction in the pK^- subsystems are parametrized using the scattering length approximation

$$F_{pK^-} = \frac{1}{1 - ika_{pK^-}}.$$

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[1]Besides the standard interpretation as $q\bar{q}$ mesons [3], these resonances were also proposed to be $qq\bar{q}\bar{q}$ states [4], hybrid $q\bar{q}$/meson–meson systems [5], or even quarkless gluonic hadrons [6].

[2]In this model we neglect the pK^+ interaction since it is repulsive and weak [15].

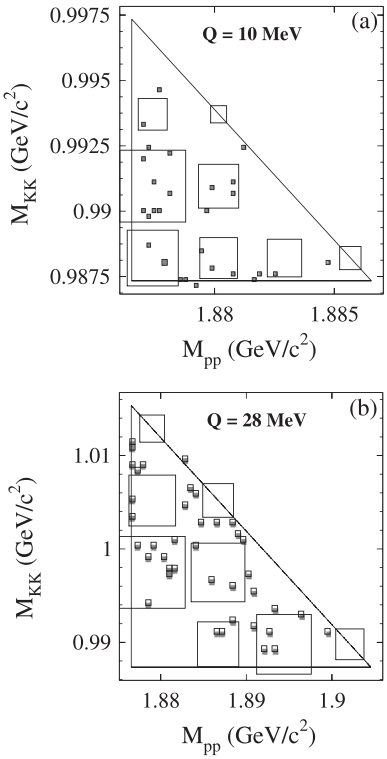

FIG. 1. Experimental Goldhaber plots for the $pp \to ppK^+K^-$ reaction. The solid lines of the triangles show the kinematically allowed boundaries. Individual events are shown in (a) and (b) as black points. The superimposed squares represent the same distributions but binned into intervals of $\Delta M = 2.5$ MeV/c^2 ($\Delta M = 7$ MeV/c^2) widths for an excess energy of $Q = 10$ (28) MeV, respectively. The area of the square is proportional to the number of entries in a given interval. The figure was adapted from [19].

The pK^- scattering length a_{pK^-} was estimated both theoretically [25–30] and experimentally based mainly on the kaonic hydrogen atom measurements [31,32]. As shown in Ref. [33] different approaches result in a slightly different a_{pK^-} values. Therefore, in our analysis we have assumed the pK^- scattering length to be equal to the mean of all the values from elaborations [25–32] summarized in Ref. [33]: $a_{pK^-} = (-0.65 + 0.78i)$ fm.

The K^+K^- FSI was parametrized using the effective range expansion,

$$F_{K^+K^-} = \frac{1}{\frac{1}{a_{K^+K^-}} + \frac{b_{K^+K^-}k_4^2}{2} - ik_4},$$

where $a_{K^+K^-}$ and $b_{K^+K^-}$ are the scattering length and the effective range of the K^+K^- interaction, respectively. We have performed a fit to the experimental data treating $a_{K^+K^-}$ and $b_{K^+K^-}$ as free parameters. Moreover, we have repeated the analysis for every quoted a_{pK^-} to check how their different values change the result. This allowed us also to estimate the systematic error due to the pK^- scattering length used in the estimation of $a_{K^+K^-}$ and $b_{K^+K^-}$.

It is worth mentioning that there is a similar phenomenological model of the K^+K^- final state interaction which takes into account the elastic and charge-exchange interaction allowing for the $K^0K^0 \rightleftharpoons K^+K^-$ transitions. This FSI should generate a significant cusp effect in the K^+K^- invariant mass spectrum near the K^0K^0 threshold (details can be found in [34]). Another contribution to this effect may be also generated by the kaons rescattering to scalars, eg.: $KK \to f_0(980) \to KK$ and $KK \to a_0(980) \to KK$. However, the ANKE data can be described well without introducing the cusp effect [34], thus we neglect it in this analysis. We also cannot distinguish between the isospin $I = 0$ and $I = 1$ states of the K^+K^- system. However, as pointed out in [34], the production with $I = 0$ is dominant in the $pp \to ppK^+K^-$ reaction independent of the exact values of the scattering lengths.

In the fit we do not take into account influence of the $f_0(980)$ and $a_0(980)$ production. There exist only very rough experimental estimates of upper limits for production of these resonances in the N-N collisions. In fact, up to now in these reactions there has not been found any signal of these particles. The theoretical estimations result in negligible cross sections for the $pp \to f_0pp \to K^+K^-pp$ resonant contribution with respect to the nonresonant one [35] (the upper limit of the cross section for this reaction is estimated to be about 1×10^{-4} nb/MeV at $Q = 5$ MeV and 4×10^{-2} nb/MeV at 50 MeV [35]). Also the branching ratios of $f_0(980)$ and $a_0(980)$ are very poorly known. However, according to the Particle Data Group (PDG) $a_0(980)$ dominantly decays to $\eta\pi^0$ and the $\pi\pi$ channel is dominant for the $f_0(980)$ meson [36]. Thus, the f_0 resonance contribution to the near-threshold $pp \to ppK^+K^-$ reaction is expected to be negligible. Moreover, regarding the $a_0(980)$ resonance, following Ref. [34] the K^+K^- pairs are produced in proton-proton collisions mainly with isospin $I = 0$. Thus, $a_0(980)$ would have to decay to K^+K^- through isospin violation, which is an additional suppressing factor. According to Ref. [35] for energies up to $Q = 115$ MeV (DISTO measurement [18]) the production of resonant K^+K^- pairs should not produce any significant enhancement in the K^+K^- invariant mass.

III. DETERMINATION OF THE K^+K^- SCATTERING LENGTH AND EFFECTIVE RANGE

In order to estimate the strength of the K^+K^- interaction the experimental Goldhaber plots, determined at excess energies of $Q = 10$ and 28 MeV, together with the total cross sections measured near the threshold were compared to the results of the Monte Carlo simulations treating the K^+K^- scattering length $a_{K^+K^-}$ and effective range $b_{K^+K^-}$ as unknown complex parameters. To determine $a_{K^+K^-}$ and $b_{K^+K^-}$ we have constructed the following χ^2 statistics:

$$\chi^2(a_{K^+K^-}, b_{K^+K^-}, \alpha) = \sum_{i=1}^{8} \frac{\left(\sigma_i^{\text{expt}} - \alpha\sigma_i^m\right)^2}{\left(\Delta\sigma_i^{\text{expt}}\right)^2} + 2\sum_{j=1}^{2}\sum_{k=1}^{10}\left[\beta_j N_{jk}^s - N_{jk}^e + N_{jk}^e \ln\left(\frac{N_{jk}^e}{\beta_j N_{jk}^s}\right)\right], \quad (2)$$

where the first term was defined following the Neyman's χ^2 statistics, and accounts for the excitation function near the

threshold for the $pp \to ppK^+K^-$ reaction. σ_i^{expt} denotes the ith experimental total cross section measured with uncertainty $\Delta\sigma_i^{\text{expt}}$ and σ_i^m stands for the calculated total cross section normalized with a factor α which is treated as an additional parameter of the fit. σ_i^m was calculated for each excess energy Q as a phase space integral over five independent invariant masses [37]:

$$\sigma^m = \int \frac{\pi^2 |M|^2}{8s\sqrt{-B}} dM^2_{pp} dM^2_{K^+K^-} dM^2_{pK^-} dM^2_{ppK^-} dM^2_{ppK^+}.$$

Here s denotes the square of the total energy of the system determining the value of the excess energy, and B is a function of the invariant masses with the exact form to be found in Nyborg's work [37]. The amplitude for the process $|M|^2$ contains the FSI enhancement factor defined in Eq. (1) and it depends on the parameters $a_{K^+K^-}$ and $b_{K^+K^-}$. The second term of Eq. (2) corresponds to the Poisson likelihood chi-square value [38] describing the fit to the Goldhaber plots determined at excess energies $Q = 10$ MeV ($j = 1$) and $Q = 28$ MeV ($j = 2$) using COSY-11 data [19]. N^e_{jk} denotes the number of events in the kth bin of the jth experimental Goldhaber plot, and N^s_{jk} stands for the content of the same bin in the simulated distributions. β_j is a normalization factor which is fixed by values of the fit parameters α and $a_{K^+K^-}$. It is defined for the jth excess energy as the ratio of the total number of events expected from the calculated total cross section $\sigma_j^m(a_{K^+K^-})$ and the total luminosity L_j [14], to the total number of simulated $pp \to ppK^+K^-$ events N_j^{gen}:

$$\beta_j = \frac{L_j \alpha \sigma_j^m}{N_j^{\text{gen}}}.$$

The χ^2 distributions (after subtraction of the minimum value) for $F_{K^+K^-}$ taken in the effective range expansion are presented as a function of the real and imaginary parts of $a_{K^+K^-}$ and $b_{K^+K^-}$ in Fig. 2. The best fit to the experimental data corresponds to

$$\begin{aligned} \text{Re}(b_{K^+K^-}) &= -0.1 \pm 0.4_{\text{stat}} \pm 0.3_{\text{sys}} \text{ fm}, \\ \text{Im}(b_{K^+K^-}) &= 1.2^{+0.1_{\text{stat}}+0.2_{\text{sys}}}_{-0.2_{\text{stat}}-0.0_{\text{sys}}} \text{ fm}, \\ |\text{Re}(a_{K^+K^-})| &= 8.0^{+6.0_{\text{stat}}}_{-4.0_{\text{stat}}} \text{ fm}, \\ \text{Im}(a_{K^+K^-}) &= 0.0^{+20.0_{\text{stat}}}_{-5.0_{\text{stat}}} \text{ fm}, \end{aligned} \tag{3}$$

with a χ^2 per degree of freedom of $\chi^2/\text{ndof} = 1.30$. The statistical uncertainties in this case were determined at the 70% confidence level taking into account that we have varied five parameters [α, $\text{Im}(a_{K^+K^-})$, $\text{Re}(a_{K^+K^-})$, $\text{Im}(b_{K^+K^-})$, $\text{Re}(b_{K^+K^-})$]. Here uncertainties correspond to the range of

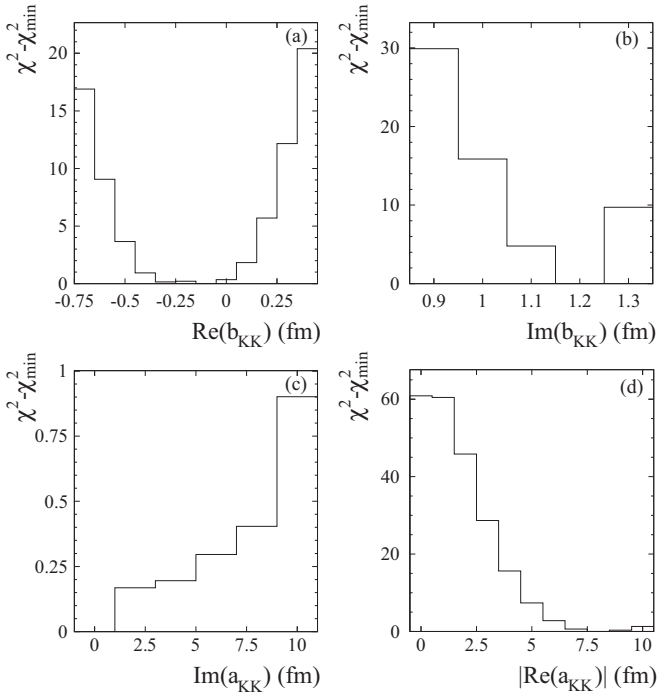

FIG. 2. $\chi^2 - \chi^2_{\min}$ distribution as a function of (a) $\text{Re}(b_{K^+K^-})$, (b) $\text{Im}(b_{K^+K^-})$, (c) $\text{Im}(a_{K^+K^-})$, and (d) $|\text{Re}(a_{K^+K^-})|$. $\chi^2_{\min}$ denotes the absolute minimum with respect to parameters α, $\text{Re}(b_{K^+K^-})$, $\text{Im}(b_{K^+K^-})$, $|\text{Re}(a_{K^+K^-})|$, and $\text{Im}(a_{K^+K^-})$.

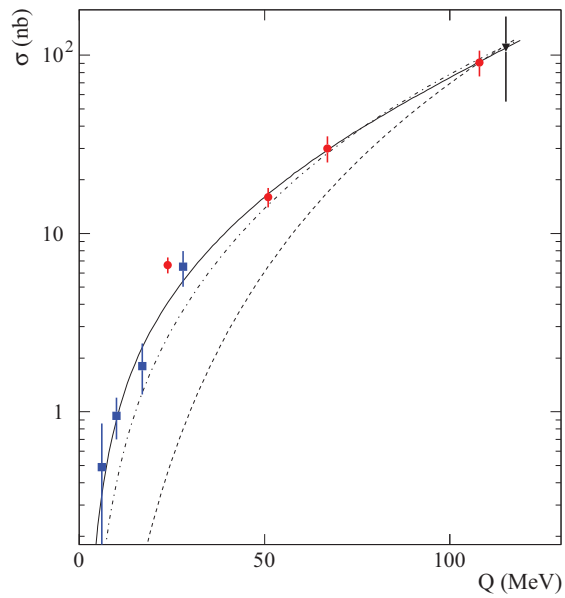

FIG. 3. (Color online) Excitation function for the $pp \to ppK^+K^-$ reaction. Triangle and circles represent the DISTO and ANKE measurements, respectively [15,18,40]. The squares are results of the COSY-11 [12,13,19] measurements. The dashed curve represents the energy dependence obtained assuming that the phase space is homogeneously and isotropically populated, and there is no interaction between particles in the final state. Calculations taking into account the pp and pK^- FSIs are presented as the dashed-dotted curve. The dashed and dashed-dotted curves are normalized to the DISTO data point at $Q = 114$ MeV. Solid curve corresponds to the result obtained taking into account pp, pK, and K^+K^- interactions parametrized with the effective range approximation. These calculations were obtained using the scattering length $a_{K^+K^-}$ and effective range $b_{K^+K^-}$ as obtained in this work. The latest data point measured by the ANKE group was published recently [40], and thus it was not taken into account in the fit.

values for which the χ^2 of the fit is equal to $\chi^2 = \chi^2_{\min} + 6.06$ [39]. Systematic errors due to the uncertainties in the assumed pK^- scattering length were instead estimated as a maximal difference between the obtained result and the K^+K^- scattering length determined using different a_{pK^-} values[3] quoted in Refs. [30,33]. One can see that the fit is in principle sensitive to both the scattering length and effective range; however, with the available low statistics data the sensitivity to the scattering length is very weak.

Results of the analysis with inclusion of the interaction in the K^+K^- system described in this article are shown as the solid curve in Fig. 3. One can see that the experimental data are described quite well over the whole energy range.

IV. CONCLUSIONS

We have performed a combined analysis of both total and differential cross section distributions for the $pp \to ppK^+K^-$ reaction in view of the K^+K^- final state interaction. In the analysis we have used a factorization proposed by the ANKE group with an additional term describing interaction in the K^+K^- system, without distinguishing between the isospin $I = 0$ and $I = 1$ states. We have also neglected a possible charge exchange interaction leading to a cusp effect in the K^+K^- invariant mass spectrum, since taking it into account would require much more precise data [34]. The K^+K^- enhancement factor was parametrized using the effective range expansion. Fit to experimental data is very weakly sensitive to $a_{K^+K^-}$, but allows us to estimate the effective range of the K^+K^-–FSI.

All studies of the $pp \to ppK^+K^-$ reaction near threshold [12–16] reveal that in the ppK^+K^- system the interaction between protons and the K^- meson is dominant, and $a_{K^+K^-}$ is relatively small. It seems that this reaction is driven by the $\Lambda(1405)$ production $pp \to K^+\Lambda(1405) \to ppK^+K^-$ rather than by the scalar mesons [34], which may, however, contribute to the observed cusp effect by rescattering of kaons. Thus, for precise determination of the kaon-antikaon scattering length we will need higher statistics, which can be available at, e.g., the ANKE experiment at COSY [41], or less complicated final states such as $K^+K^-\gamma$ or $K^0\overline{K^0}\gamma$, where only kaons interact strongly. These final states can be studied for example via the $e^+e^- \to K^+K^-\gamma$ or $e^+e^- \to K^0\overline{K^0}\gamma$ reactions with the KLOE-2 detector operating at the DAΦNE ϕ factory [42].

[3]Due to the fact that in the case of scattering length the systematic uncertainties are much smaller than the statistical ones we neglect them in the final result.

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